Exercise 2.

Time Domain analysis

Required knowledge

- Time-domain description of first- and second-order systems.
- Measurement of phase shift between periodic signals.
- Transmission line theory: reflection calculation, wave propagation.
- Theory of averaging of noisy signals.

Introduction

Time domain investigation of signals and systems is one of the most essential tool of electrical engineering. When a physical phenomenon is investigated, its time domain behavior is one of the most important property which should be observed. In infocommunication often the shape of the received signal carries the information (e.g., its amplitude, phase, rate of change…). Even if a signal is stored or transmitted in digital form, most essential building blocks of digital signals (bits) are represented by analogue signals in the physical layer. In order to establish a high quality digital communication, the analogue signals must be well-conditioned: high signal-to-noise ratio should be achieved, the state transitions should be sharp enough, oscillation and reflections should be avoided.

Simple first-order systems and transmission lines that will be investigated in the measurement are basic building blocks of several complex systems, so it is crucial to be familiar with the time-domain behavior and measurement technique of these systems.

Aim of the measurement

Students will perform the following task: (1) time- and phase measurement, (2) frequency dependent transfer of linear systems, investigation in time domain, (3) signal shaping in distributed parameter systems, (4) averaging as noise suppression. They will get acquainted with time domain reflectometry, and practice the time and phase measurement with oscilloscope, and failure diagnosis by means of investigation of time domain waveforms.

Web links

http://en.wikipedia.org/wiki/Lissajous_curve

http://en.wikipedia.org/wiki/Time-domain_reflectometry

Measurement instruments

Power supply	Agilent E3630A
Function generator	Agilent 33220A
Oscilloscope	Agilent 54622A
Multimeter	

Theoretical background

Measurement of pulse parameters

The definition of pulse parameters are given in Figure 2-1:

- Rise-time: the time during which the signal increases from the 10% to 90% of the final value. Care should be taken, since the base point is at the low level of the signal. For example, if U_{low} =1 V and U_{high} =10 V, then threshold values are: $U_{10\%}$ = 1.9 V and $U_{90\%}$ = 9.1 V.
- Fall time: the time during which the signal decreases from 90% to 10% of the initial value. 90% and 10% again refers to the difference between U_{low} and U_{high}.
- Overshoot: the difference between the peak value and the final value of the signal. It is often given relative to the final value in percent.
- Undershoot: the difference between the negative peak value and the final value of the signal at the falling edge.
- Droop: the decrease of the amplitude of the pulse from the beginning to the end.
- Impulse width: the time difference between the 50% threshold levels of the positive and negative edges.
- Settling time (ringing time): the time during which the signal settles after the level transition at its input within a specified interval around the final value of the signal (and doesn't leaves this interval any more). The typical values of specified interval are, e.g., ±0.1%, ±1%, ±5% around the final value.

These methods are based on graphical evaluation, hence the measurement of the parameters is sometimes not obvious and not well-defined (e.g., at wrong signal-to-noise ratio, spurious oscillations occur…).

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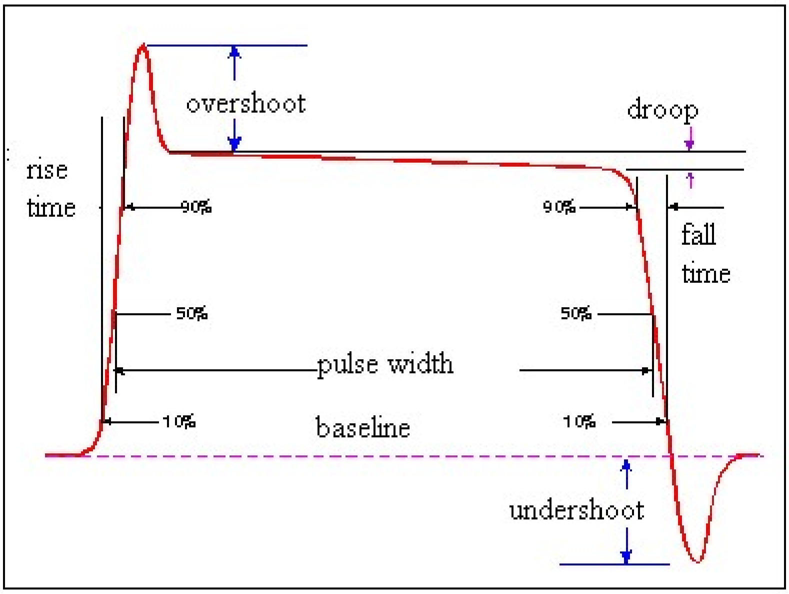

Figure 2-1. Definition of pulse parameters

It is a general rule that either of the previous parameters are measured, the range (both time/div and volt/div) should be as tight as possible, i.e., we should zoom on the measured part of the signal as close as possible in order to minimize the measurement error.

Modern digital oscilloscopes are able to measure these parameters automatically (Quick Measure button on the oscilloscopes used in the laboratory). However, the ranges should be set manually before these functionalities are used, and measurements should be verified visually, since these automatic measurements are based on the data which are displayed on the screen of the oscilloscope. For example, if rise time is measured and the time/div setting is too high, then the rising edge may be seen as 1-2 pixels on the screen. In this case even the oscilloscope can not do precise measurement. Contrary, if the time/div is too fine, and the steady-state high and low levels can not be seen on the screen (we zoom too close to the edge, and other parts of the signal can not be seen), then the oscilloscope cannot correctly calculate the 10% and 90% threshold levels, so the measurement will be incorrect. Quick Measure function is a useful tool, however, it is recommended to make some measurements manually, otherwise we won't know how to set up the oscilloscope for the measurements.

First-order RC circuits

During the course of laboratory measurement first-order, low- and high-pass filters will be investigated. One of the simplest measurement method is the measurement of the step response of the systems which can be performed with a simple square wave generator. The analytical form of the step responses of general first-order, low- and high-pass systems are:

$$v_{\mathrm{LP}}(t) = A(1 - e^{-\frac{t}{\tau}}), \qquad v_{\mathrm{HP}}(t) = Ae^{-\frac{t}{\tau}}, \tag{2-1}$$

where A is the amplitude gain and τ stands for the time constant of the system. In the laboratory, first-order RC filters will be investigated whose schematic diagrams are shown below:

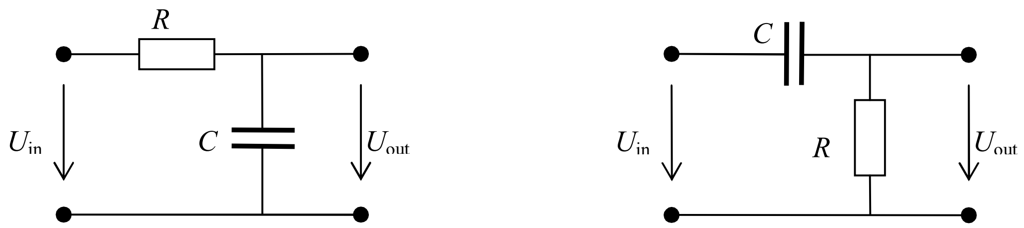

Figure 2-2. Schematic diagrams of first-order, low- and high-pass RC networks.

The time constant of such systems is $\tau = RC$, and their gain is unity: $A = 1$. The responses of these systems on a step function of amplitude U_{peak} are:

$$v_{\mathrm{LP,RC}}(t) = U_{\mathrm{peak}}(1 - e^{-\frac{t}{RC}}), \qquad v_{\mathrm{HP,RC}}(t) = U_{\mathrm{peak}} e^{-\frac{t}{RC}}, \qquad (2\text{-}2)$$

Step response of the first order RC networks are shown in the figures below:

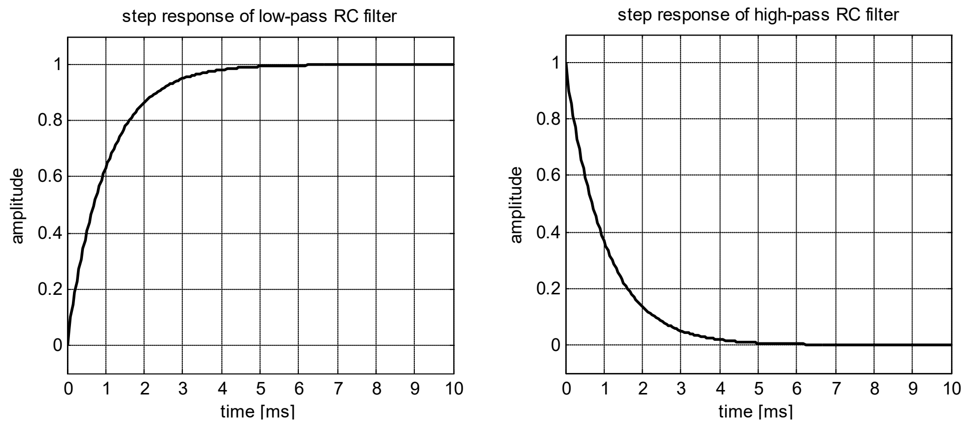

Figure 2-3. Step responses of first-order, low- and high-pass RC networks. Time constant is: $\tau = 1/\omega_0 = RC = 1$ msec.

Measurement of time constant of first-order systems

The time constant of the systems will be measured using square wave input signals. If the half of the period of the square wave used as excitation signal is considerably longer (at least 5 or 10 times) than the time constant of the system to be measured, the square wave can be regarded as a periodic step function, and the output of the system can be regarded as the step response of the system. The measurement arrangement is found in the figure below. First-order RC circuits contain only passive components so they do not require supply voltage.

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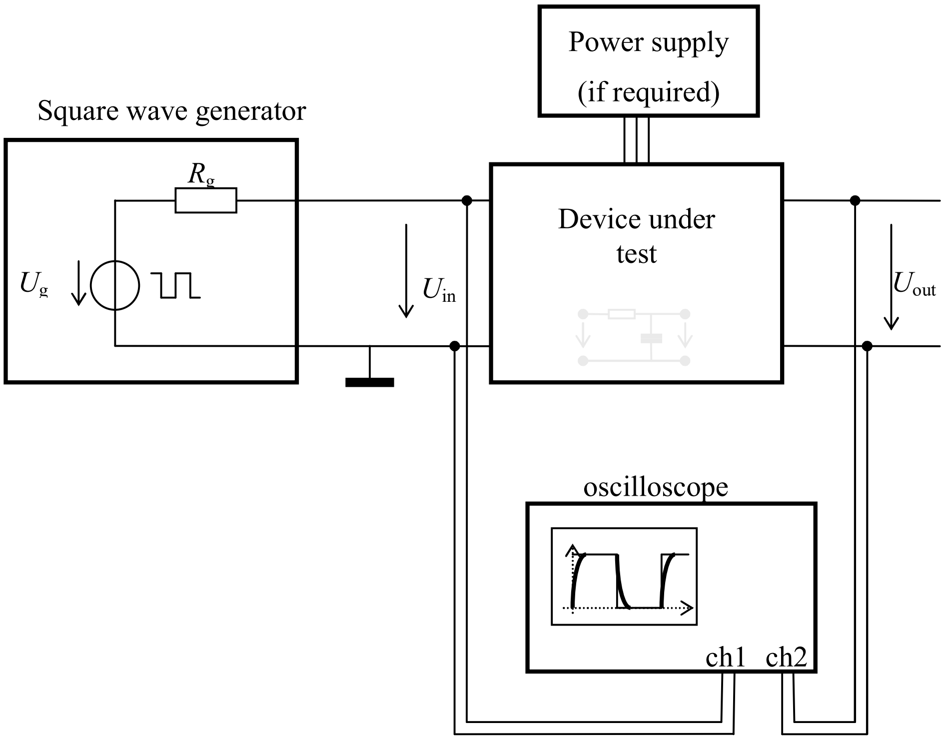


Figure 2-4. Block diagram of the step response and time constant measurement.

In the figure, R_g denotes the output impedance of the function generator ($R_g = 50\ \Omega$). This resistance has practical significance if the input impedance of the DUT is not considerably higher than R_g. In this case, the input signal can be less than the value set on the function generator since the input impedance of the DUT and R_g form a voltage divider. R_g can also influence the time constant of the system, since it is added to the resistance of the RC network.

Three methods will be introduced to measure the time constant based on the step response. These methods are summarized in the figures below:

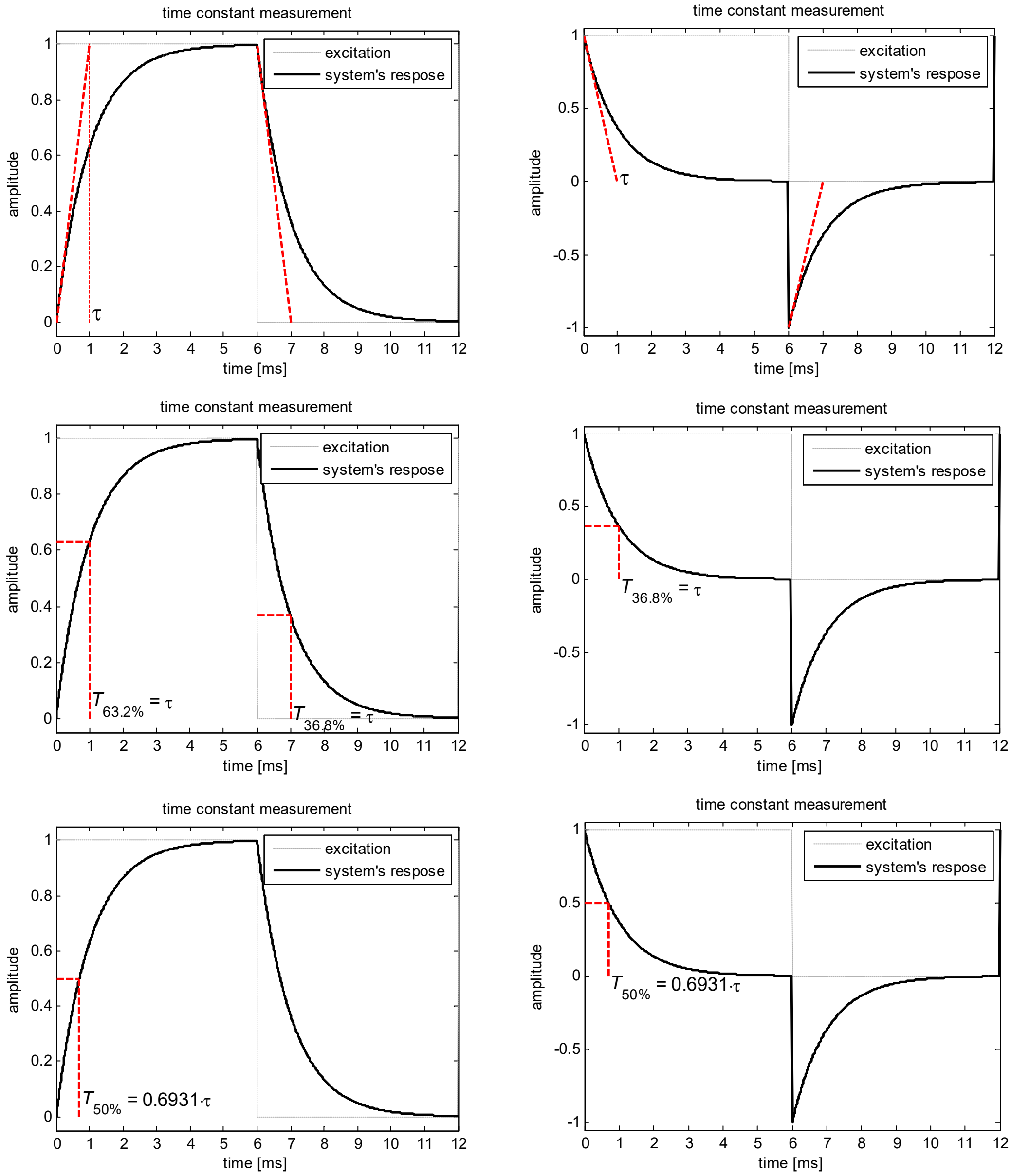


Figure 2-5. Illustrations of time constant measurement methods based on step response. Left column: low-pass filter; right column: high-pass filter. First row: tangent at zero point; second row: measurement at the 63.2% and 36.8% of the maximum value; last row: measurement at the 50% of the maximum. The time constant in this example is $\tau = 1$ ms.

In the examples, the half of the square wave is more than five times the time constant so the system's response achieves the steady state before each new edge of the excitation signal.

The methods of time constant measurement are:

1. Time constant measurement based on tangent at zero point:
   - At the falling edge: draw the tangential of the step response at the beginning of the falling edge. The tangential crosses the time axis at the time constant.

- At the rising edge: draw the tangential of the step response at the beginning of the rising edge. The tangential reaches the final value of the step response at the time constant.

2. Measurement at the 63.2% and 36.8% of the maximum value:
   - low-pass filter: the step function reaches the 63.2% of the final value after rising edge, and it reaches the 36.8% of the initial value at the falling edge after the time constant. Note that 63.2% = 1–1/*e* and 36.8% = 1/*e*.
   - high-pass filter: the step function reaches the 36.8% of the initial value after the time constant (i.e., it decreases to the *e*-th part during the time constant).
3. Measurement at the 50% of the maximum value: the step function reaches its 50% after 0.6931 times the time constant. Note that 0.6931 = –log(1–1/2).

All of these methods can be proved according to the step response of first-order systems given in equation (2-1).

To prove the first method, the derivative of the step response should be calculated, that is $v'_{\mathrm{LP}}(t=0) = \frac{A}{\tau} e^{-\frac{t}{\tau}} = \frac{A}{\tau}$ for low-pass filter. Since $v_{\mathrm{LP}}(t=0) = 0$, the tangential reaches the amplitude A after at $t=\tau$. Proof is similar for high-pass filter (it should be solved as homework).

The proof the second and third method differs only in the last step. The final value of the step response of low-pass filter is A. In order to calculate how many time it takes to reach a value aA, we should solve the equation $v_{\mathrm{LP}}(t) = A(1-e^{-\frac{t}{\tau}}) = aA$. One obtains that it is true for $t = -\tau\cdot\ln(1-a)$. For $a = 63.2\% = 1-1/e$ one obtains $t = -\tau\cdot\ln(1-(1-1/e)) = \tau$, and for $a = 50\% = 0.5$ one obtains $t = -\tau\cdot\ln(1-0.5) = 0.6931\cdot\tau$. Proof is similar for high-pass filter (it should be solved as homework).

Measurement of the transfer function

It is well known that a linear time-invariant system can change only the phase and amplitude of a sine wave applied to its input. Hence, the system can be characterized at each frequency by a complex number (complex gain) whose phase is the phase shift of the system, and its magnitude is the gain of the system. The transfer function is the complex gain of the linear system as function of frequency.

Several methods are known which allow the measurement of the transfer function of linear systems. In the following, some of these methods are summarized (the emphasis is put on the measurement of magnitude characteristics).

Measurement of amplitude characteristics with stepped sine

A well-known method of measurement of amplitude characteristics is performed using a sine wave generator and an AC multimeter (Figure 2–6). The measurement doesn't require expensive special instruments if high precision is not crucial. Its disadvantage is that the measurement is relatively time consuming, since the amplitude characteristics should be measured point-by-point along the whole frequency range. The frequency resolution of the measurement is determined by the frequency resolution of the sine wave generator. When only the bandwidth is to be measured, it can be done by setting the frequency to the center frequency where the gain is nominal, and then the frequency should be changed until the output signal decreases by 3 dB. The multimeter can often be exchanged with an oscilloscope, but the precision of an oscilloscope is generally worse than that of a multimeter.

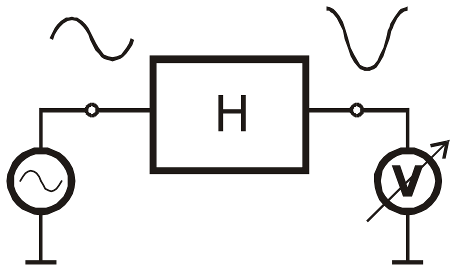


Figure 2–6. Measurement of transfer function with sine wave generator and multimeter

The amplitude reference point has to be set before beginning the measurement. Every subsequent measurement result is compared to this reference point. The reference point is set according to the type of the amplitude characteristics (e.g., high-pass, low-pass, band-pass…). For example, if a system has low-pass characteristics as shown in the figure below, the reference point should be set at low frequency, at least one or two decades below the cutoff (corner) frequency. If the multimeter has fixed 0 dB point, it is recommended to set the input signal such that 0 dB appears at the output. Some of the modern multimeters allow us to set the 0 dB point to an arbitrary value. In this case, the input signal should be set as high as possible in order to ensure good signal-to-noise ratio. Care should be taken when setting the level of input signal! A common mistake is that the output signal becomes distorted, e.g., due to saturation, or the measured values are out of the range of the instruments. Except of some special cases, neither the input nor the output signals can exceed the supply voltage. If a passive circuit is measured (e.g., first-order RC network), no power supply is required. The level of input signal shouldn't be changed during the whole measurement. It is generally recommended to check the shapes of the signals with an oscilloscope during the measurement.

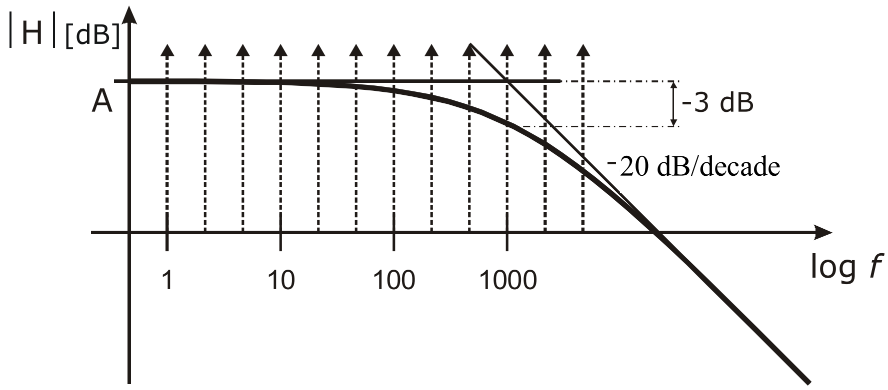


Figure 2–7. Transfer function of a low-pass filter

During the course of the measurement, the frequency is often changed logarithmically (see Figure 2–7), e.g., with steps 1-2-5-10-..., but it is recommended to measure with finer steps in the vicinity of the cutoff frequency. The cutoff frequency is often defined as the frequency where the amplitude characteristics decreases by 3 dB below the nominal value. (E.g., if the nominal gain is 9 dB, the gain is 6 dB at the cutoff frequency.)

The stepped sine wave method has the advantage that it offers a good signal-to-noise ratio. However, the measurement of the whole amplitude characteristics requires considerable time, since the frequency should be changed after each measurement, and we should wait until the transient vanishes after each time the frequency is changed.

Measurement of phase difference

The method used in this measurement is traced back to the measurement of ratio of time intervals, more precisely on the ratio of the time delay between two signals and the period of the signal. The method is illustrated in Figure 2-8. The two signals are fed to the two different channels of the oscilloscope. Then we should search the same reference points on the two signals. Practically, the positive or negative zero crossing points are used as reference points. Let Δt denote the time delay between these two reference points. Furthermore, let T denote the period which can be measured as the time difference between two consecutive positive or negative zero crossing of the signal. The phase difference can be calculated as $\varphi = \frac{\Delta t}{T} \cdot 360°$. The advantage of the method is that it is not sensitive to the time base error of the oscilloscope, only the linearity of the time base is required. However, it is true only until the time/div setting remains the same when Δt and T are measured. If Δt is considerably smaller than T, then Δt should be measured with smaller time/div setting (finer time resolution). In this case the error of time base can not be neglected when measurement error is calculated. It depends on the specification of the oscilloscope whether Δt and T should be measured with the same or different time base setup.

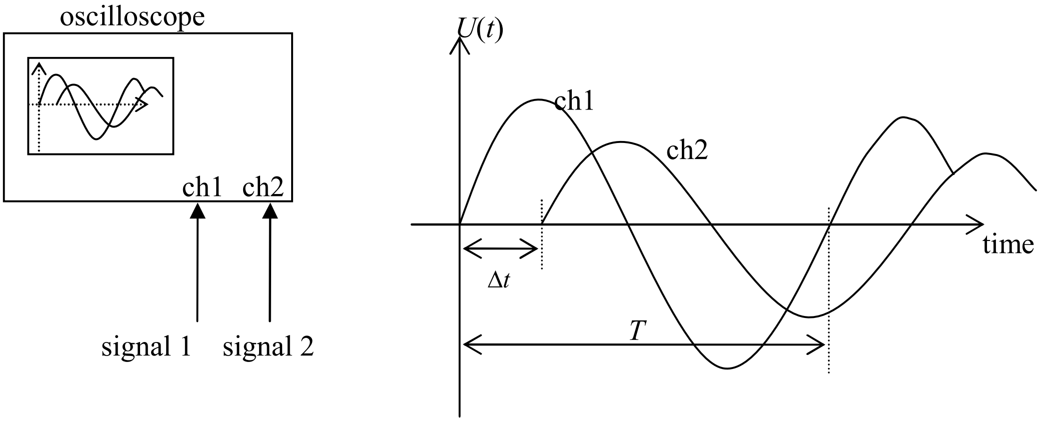


Figure 2-8. Phase difference measurement based on time interval measurement

The oscilloscope used in the laboratories have built-in phase shift measurement functionality which is based on the previous method. This tool works properly only when at least one whole period of the observed signal can be seen on the display of the oscilloscope. This constraint limits the accuracy when small phase difference is measured, since in this case we can not zoom into the time difference (Δt) between the signals which would required to make a precise measurement. This example shows that it is highly recommended to be able to perform manual measurement since automatic functionalities may fail in some cases.

Transfer function of first-order systems

The transfer function of the first-order low-pass (W_{LP}) and high-pass (W_{HP}) filters of Figure 2-2. can be written as:

$$W_{\mathrm{LP}} = \frac{1}{1 + j\omega/\omega_0}, \quad W_{\mathrm{HP}} = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}. \tag{2-3}$$

In electrical engineering, the piecewise linear approximations of Bode plots are also often used. These plots are given for first-order systems in figures below:

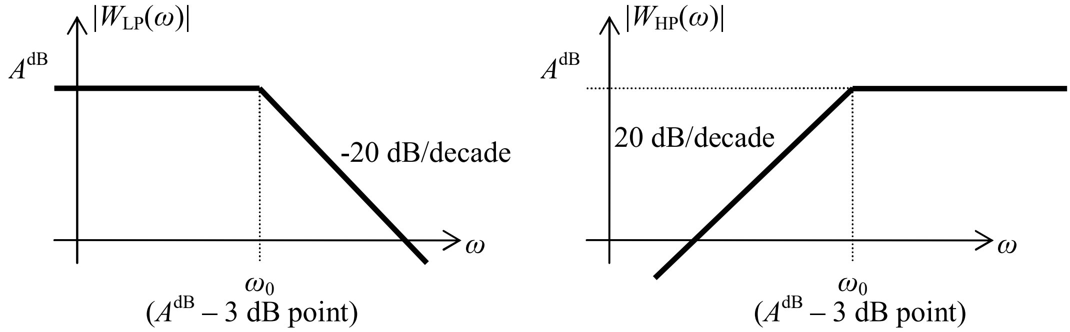


Figure 2–9. Piecewise linear approximation of Bode plots of first-order, low-and high-pass systems.

The precise transfer functions calculated by MATLAB are:

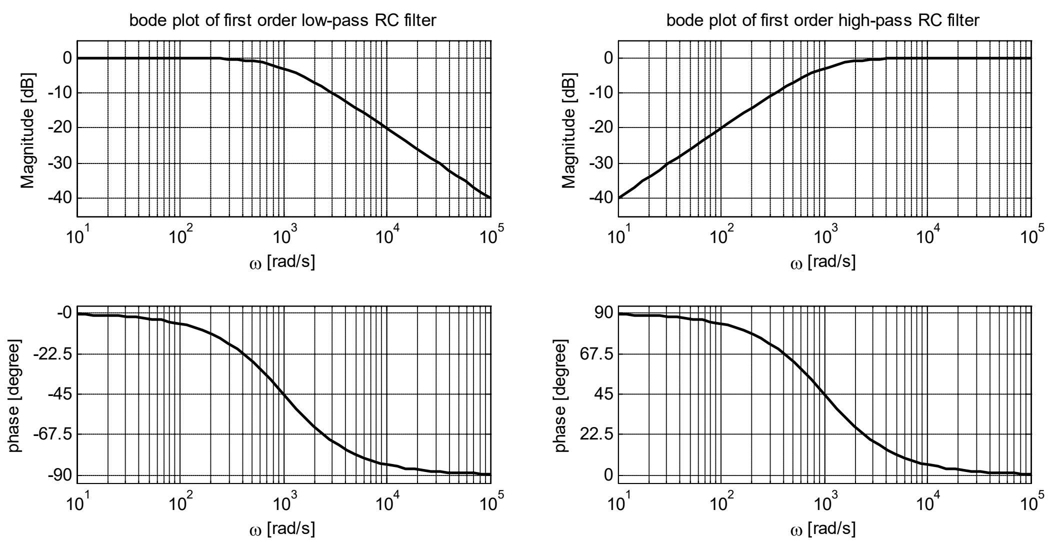


Figure 2–10. Transfer functions of first-order, low- and high-pass RC networks. Cutoff frequency is in this example: $\omega_0=1/RC=1000$ rad/sec.

Exercise 2.

Important properties of these networks are:

property	low-pass RC network	high-pass RC network
Cutoff frequency / time constant	$\omega_0 = 1/\tau = 1/RC$	$\omega_0 = 1/\tau = 1/RC$
DC gain	0 dB (1)	-∞ dB (0)
gain at cutoff frequency	-3 dB ($1/\sqrt{2}$)	-3 dB ($1/\sqrt{2}$)
gain at $\omega \to \infty$	-∞ dB (0)	0 dB (1)
slope of $\lvert W \rvert$ below cutoff frequency	0 dB/decade	20 dB/decade
slope of $\lvert W \rvert$ above cutoff frequency	-20 dB/decade	0 dB/decade
DC phase shift	0°	90°
phase shift at cutoff frequency	45°	45°
phase shift at $\omega \to \infty$	90°	0°

The knowledge of basic behavior of low- and high-pass networks is also important when instruments are characterized. For example, when an oscilloscope is used with AC coupling, its input stage behaves like a high-pass filter. The cutoff frequency of AC coupling of the oscilloscope Agilent 54622 is 3.5 Hz by specification.

For high-frequency signals an instrument (oscilloscope, mulimeter…) behaves like a low-pass filter. Care should be taken, since not only the fundamental, but higher order harmonic components can be modified by the instrument. For example, if the bandwidth of an oscilloscope is 100 MHz, and a periodic square wave of 10 MHz is measured, the effect of the oscilloscope's bandwidth even on the 10-th (and higher order) harmonic components can not be neglected. It will result in the phenomenon similar as if the square wave would be composed of only harmonic components up to the order of ten, hence sharp edges will disappear. The bandwidth of some oscilloscopes can also be decreased intentionally to improve signal-to-noise ratio when low frequency signals are measured.

Time-domain reflectometry (TDR)

Transmission lines are often used in micro-wave circuits, impulse technique and high-speed digital systems. Every conductor can be regarded as transmission line if its length is at least approximately the tenth of the wavelength of the signal to be transmitted. If high frequency signal is transmitted through a conductor, the nature of the propagation of electromagnetic waves in transmission lines should be considered.

The following figure shows a block diagram where time-domain reflections in transmission lines can be investigated. A voltage generator is used which is able to provide a step function at its output (in practice a square wave generator is used with long period such that steady state is achieved between level transitions). The output impedance of the generator equals to the wave impedance of the transmission line, i.e. $Z_g = Z_o$ (matched termination on the input). The voltage $e_x(t)$ is measured with an oscilloscope at the output of the generator that is connected to the input of the transmission line. The transmission line is terminated with a real valued load with impedance of value Z_L.

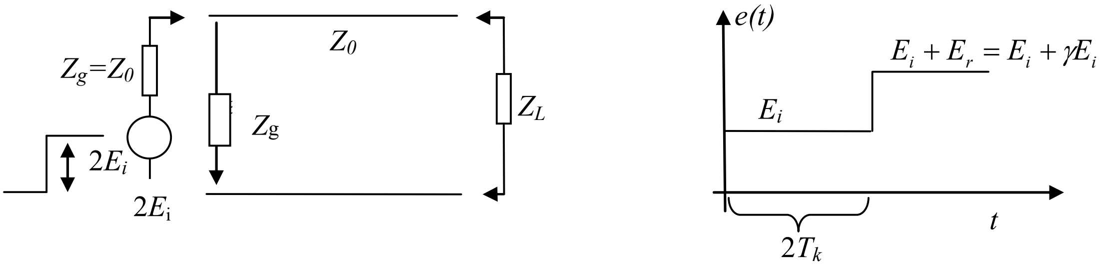


Figure 2-11. Time domain reflectometry: Block diagram and time-domain signals for a step function input

Let us estimate the signal shapes with simple physical considerations. The system is in idle state before time instant $t = 0$. When the step function appears at the input, the transmission line shows a wave impedance of Z_o independently on the load impedance. The reason is that the signal has finite propagation speed, so the signal "doesn't know" when it is appeared on the input what are the load conditions; it "sees" only the wave impedance of the cable. Hence, a voltage divider is formed from the generator impedance Z_g and wave impedance Z_o, and a step wave of amplitude E_i propagates towards the end of the end of the cable. When the wave reaches the load impedance (after a time T_k), a reflection occurs. The reflection coefficient (γ) depends on the load impedance and wave impedance of the cable as the following equation:

$$\gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0}. \tag{2-4}$$

The reflected wave is γ times of the incident wave, i.e., it is $E_r = \gamma E_i$. The voltage on the load is zero until a wave reaches this point (until time instant T_k), and after the time instant T_k the sum of the incident and the reflected voltage (E_i+γE_i) is measured. The voltage observed on the input remains E_i until the reflected wave arrives back to the input, and than it becomes (E_i+γE_i). Since we investigate the case when $Z_g = Z_o$, so no more reflection occurs on the input, hence the steady state has been achieved.

The propagation time from the input to the end of the cable is denoted by T_k. The round-trip delay during which the first reflection (E_r) arrives at the input of the cable is $2T_k$. If the input is matched, i.e., $Z_g = Z_o$, then the steady state has been achieved, and the steady-state input voltage is $U_{ss} = E_i + \gamma E_i$. Substituting γ into this equation, and using that $Z_g = Z_o$, one obtains:

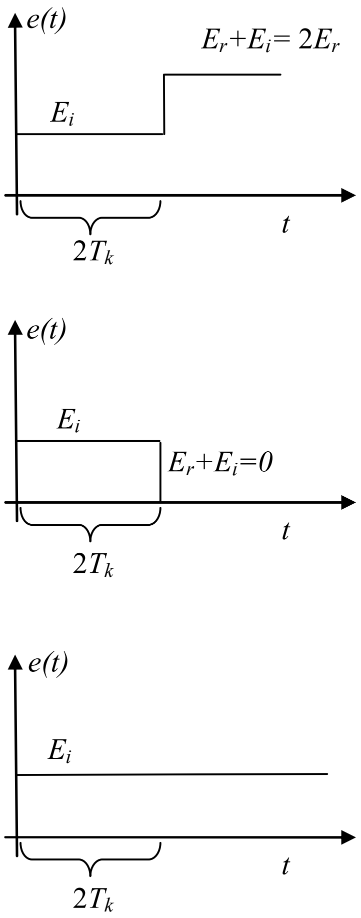
$$U_{ss} = 2E_i \frac{Z_L}{Z_L + Z_g}, \tag{2-5}$$

which means that the steady-state voltage can be calculated as if the load impedance were directly connected to the generator (note that the amplitude of the input signal is $2E_i$).

The above described measurement is called TDR (Time-Domain Reflectometry). This kind of measurement can be used to detect whether a cable is terminated correctly (no reflection occurs.

Exercise 2.

Some important case is illustrated in the following figure:



$Z_L = \infty \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = +1$ *a)*

$Z_L = 0 \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$ *b)*

$Z_L = Z_0 \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$ *c)*

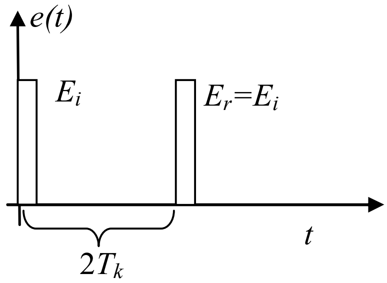
Figure 2-12. Waveforms as a response to step function input, measured at the input of the transmission line for different loads: (a) open circuit, (b) short circuit, (c) matched load

Case (b) is important when short pulses have to be generated, since the pulse duration can be tuned with the length of the cable.

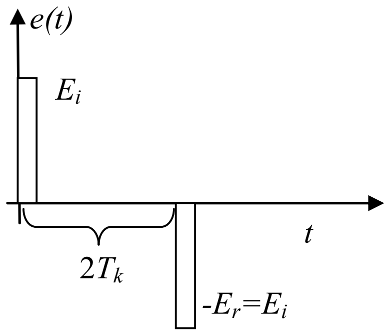
Note that if the generator impedance isn't matched ($Z_g \neq Z_o$) too, reflection happens also on the input, and all of the previously described rules can be applied to calculate reflection on the input.

If the load impedance is not real-valued, then the waveforms are more complex. The waveforms in initial state ($t = 0$) can be approximated by substituting $Z_C \to 0$; $Z_L \to \infty$, and in steady state ($t = \infty$) conditions $Z_L \to 0$; $Z_C \to \infty$ can be used. In the intermediate states the waveforms are exponential depending on the nature of the load.

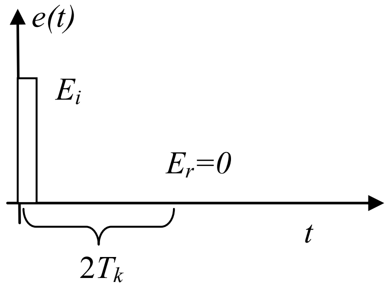
If we can generate a very narrow pulse (the generator used in the lab can do this), we can get an even clearer picture of signal propagation. If the length of the pulse is shorter than the back and forth propagation time on the cable, the incident and reflected components do not add up as in the case of the step response, but are well separated. Some special cases are illustrated in the following figure.



$$Z_L = \infty \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = +1 \qquad a)$$



$$Z_L = 0 \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 \qquad b)$$



$$Z_L = Z_0 \qquad \gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \qquad c)$$

Figure 2-13. Reflected signal waveforms for pulse excitation with (a) open-circuit, (b) short-circuit and (c) matched load, measured at the input of the transmission line.

TDR is used to investigate long cables. Any kind of damage influences the wave impedance of the cable, which causes reflection at the position of the damage. To perform a TDR measurement, we need to access only one end of the cable. The waveforms allow us to predict the type of damage (short circuit: $Z_L \to 0$, break $Z_L \to \infty$). Timing values allow us to predict the location of the damage. By multiplying the half of the round-trip delay (T_k) by the propagation speed of the wave (v), one obtains the location of damage: $l = v \cdot T_k$. The propagation speed of the wave depends on the dielectric constant $v = \frac{c}{\sqrt{\varepsilon_r}}$, where c is the speed of light in vacuum and ε_r is the relative dielectric constant of the material.