

Exercise 6

Two-terminal Electronic Components

Required knowledge

Students are assumed to be acquainted with the followings:

- basic knowledge about the simplest two-terminals (R,L,C),
- temperature dependence of resistors,
- series and parallel LC circuits, the resonance phenomenon,
- impedance measurement methods,
- drawing and understanding Bode diagrams of two-terminals built from two or three building elements (both amplitude and phase).

Introduction

Passive two-terminal components (R, L, C) are the simplest building elements of both circuit modeling and practical systems. However, two-terminal components utilized in practice can only approximately be considered as ideal (e.g., coil wires have resistance, iron cores have loss, capacitors have both resistance and inductance), hence giving precise descriptions is possible only using more complex models. Getting familiar with the properties of two-terminals found in real systems is crucial in order to be able to design and analyze electronic circuits. On the other hand, sometimes the aim is to realize a combined two-terminal to achieve the desired behavior. (E.g., in an equalizer the filters having the desired frequency response can be realized with combined two-terminals.) Getting acquainted with measurement methods and measuring equipment lets us plan measurements, with which we can obtain more detailed information about the devices being analyzed and can point out the limitations and the errors of the applied methods and equipment.

Aim of the measurement

There are three main aims of the measurement: (1) study the non-ideal properties of RLC elements; (2) understand the complex frequency-dependent behavior of combined two-terminal components and determine their corresponding parameters with various measurements; (3) study the applied methods and their limitations. These measuring exercises improve the ability to produce models, and to show the errors of real measurements and devices. The students also practice the analysis of electronic circuits.

Theoretical fundamentals of the measurement

Impedance

Impedance is one of the commonly used parameters to describe components, materials, or electronic circuits. Impedance (\bar{Z}), by definition, is a complex quantity that expresses the resistance of a given equipment against the current flowing through it. An impedance can be characterized by a complex phasor, which consists of a real and an imaginary part:

$$\bar{Z} = R + jX, \quad (6-1)$$

where R and X denote the resistance and the reactance, respectively.

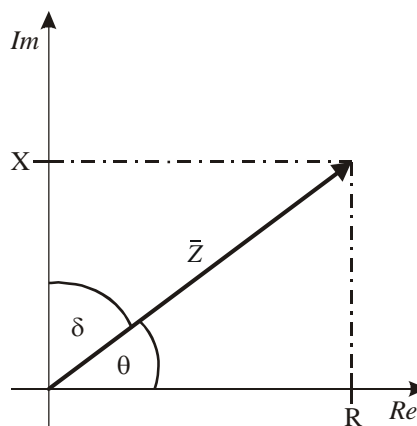


Figure 6–1 Complex phasor of an impedance

If the reactance of a component is positive, then we call it inductive, if it is negative, then we call it capacitive, and if it is zero, then we call it purely resistive. We expect an inductor to be inductive and have a reactance ($X = X_L$) proportional with frequency, and we expect a capacitor to be capacitive and have a negative reactance ($X = -X_C$) inversely proportional with frequency, as indicated by the following formulas:

$$X_L = \omega L = 2\pi f L, \quad (6-2)$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}, \quad (6-3)$$

where f denotes frequency, ω is the angular frequency, L is the inductance, and C is the capacitance. Note that the value of L and C can also be a function of parameters like voltage, current, frequency, time, temperature, etc.

Impedance can also be characterized by the quality (Q) or the dissipation (D) factor. Quality factor is the quotient of the reactive power and the real power, whilst dissipation factor is its reciprocal value. Both parameters are dimensionless, dissipation factor is commonly given in percentage. For characterizing inductors mainly quality factor is used, while for capacitors the dissipation factor is more common. As a rule of thumb, the greater the quality or the

smaller the dissipation factor, the better the electrical component. Dissipation factor is often called $\tan \delta$, where δ is the angle of the dielectric and not the angle of the impedance (see Fig. 6–1). Since reactance is frequency-dependent, quality and dissipation factors are frequency-dependent parameters, as well, and Q and D values are usually given at certain frequencies or for frequency ranges in the datasheets.



Figure 6–2 Series impedance models

For series RL and RC models these parameters can easily be calculated:

$$Q = \frac{|X|}{R} \Rightarrow Q_L = \frac{\omega L}{R}, Q_C = \frac{1}{\omega RC}, \quad (6-4)$$

$$D = \frac{R}{|X|} \Rightarrow D_L = \frac{R}{\omega L}, D_C = \omega RC. \quad (6-5)$$

For parallel RL and RC models they become:

$$Q = \frac{R}{|X|} \Rightarrow Q_L = \frac{R}{\omega L}, Q_C = \omega RC, \quad (6-6)$$

$$D = \frac{|X|}{R} \Rightarrow D_L = \frac{\omega L}{R}, D_C = \frac{1}{\omega RC}. \quad (6-7)$$

Impedance measurement

Parameters to be measured

In the simplest case impedance measurement consists of determining the parameter of a one-parameter model (R, L, or C). However, in this case the behavior of the model is usually significantly different from that of the object being measured, and this error cannot be reduced under a certain limit even if the measurement is perfect. This phenomenon is the modeling error. To obtain more precise results, a model with more parameters is needed. A complex impedance at a certain frequency can be characterized with a complex quantity, where the real and the imaginary parts of the complex impedance (or its reciprocal, the admittance) correspond to ideal two-terminals. Thus more precise impedance meters always measure parameters of a two-parameter model at a given frequency. Using a two-parameter model is still a simplification. Two-parameter models of impedance measurement always contain only one dynamic element, so they cannot model resonance. Contrarily, real passive elements mostly have resonance (\sim LC circuit).

During impedance measurement any equivalent circuit model parameters of any impedance can be measured, or, more precisely, derived (e.g., an inductor can be modeled as a parallel RC component, but in this case the result of the measurement would probably be a negative capacitance). However, the given equivalent circuit model does not give any general information about the frequency domain behavior, it only provides correct results at the given frequency. The properties of the object can be determined using measurements in a wider frequency range.

Measurement methods

Impedance analyzers usually utilize complex ratio measurement as a measurement principle. During this, the object being measured (\bar{Z}_x) and a precision impedance (usually resistor, R_s) are connected in series and the same current flows through them. Utilizing the ratio of the voltage drops on the object being measured and on the precision resistor, the unknown complex impedance \bar{Z}_x can simply be determined:

$$\bar{Z}_x = R_s \frac{\bar{U}_x}{\bar{U}_s}. \quad (6-8)$$

(Note that in the above expression voltages are complex quantities, hence the resulting \bar{Z}_x also contains the phase information.)

The most critical part of the measurement is the connection between the object and the analyzer. Different measurement settings (two-, three-, four-, and five-wire measurements) enable us to reduce the effects of parasitic elements. These arrangements are also often referred to as two-terminal, three-terminal, etc.

The simplest way to connect the object to the analyzer is the so called two-wire measurement. In this case, however, not only the impedance of the object, but also the impedance (which can be considered as resistance, especially at low frequencies) of the connecting cables is included in the measured result – see Fig. 6–3(a).

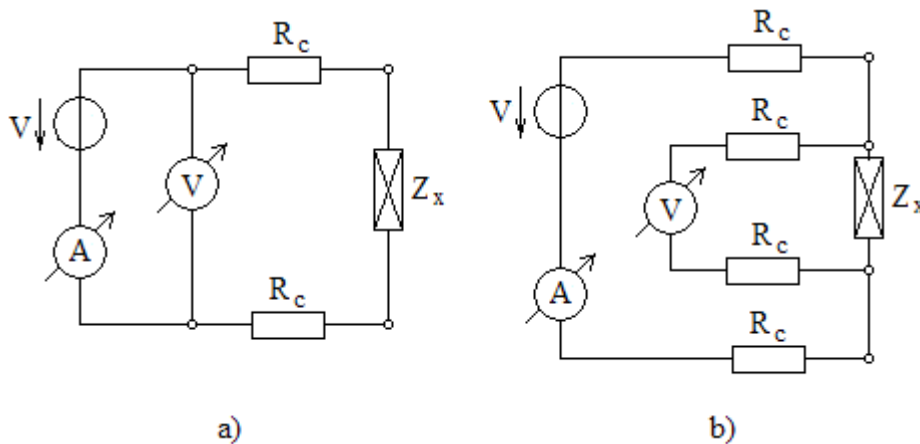


Figure 6–3 Two- (a) and four-wire (b) measurements

Using the four-wire measurement technique two more cables are needed, but the impedance of the cables does not distort the result, considering the following. The impedance of the voltage meter is much higher than that of the object being measured (ideally it would be infinite). Due to this, approximately no current flows through the voltage meter, which means that no voltage drop occurs, either. So the voltage meter measures the voltage drop

only on the object being measured. And since no current flows through the voltage meter, the current meter measures the current on the object. Paradoxically, to reduce the negative effect of the cables, two more of them are needed!

Note that the four-wire measurement technique does not eliminate all parasitic effects, since stray capacitances are still there in the circuit. Three-wire measurement technique eliminates them, but has no effect on cable resistances, while five-wire measurement solves both issues.

Resistors

Resistors are components having a certain resistance. Remember that the ‘resistor’ is the component, while ‘resistance’ refers to the physical quantity – which we can calculate with. There are two main resistor groups: linear and non-linear resistors. Resistors can be characterized with the following parameters:

- nominal value,
- tolerance,
- temperature coefficient,
- maximum load (maximum power),
- quality class.

Nominal resistor values are given by standards. It would obviously not be wise to produce arbitrary resistor values, that is why values of resistor series have been standardized. Standard resistance series are referred to as E_x, where *x* is a number that refers to the number of different resistor values within a decade.

E6	1		1.5		2.2		3.3		4.7		6.8	
E12	1	1.2	1.5	1.8	2.2	2.7	3.3	3.9	4.7	5.6	6.8	8.2

Resistor values are mostly marked by the international color code notation system.

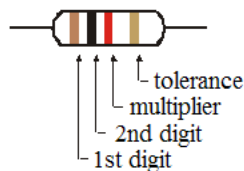


Figure 6–4 International four-band color code notation sytem

Capacitors

Capacitors are components having a certain capacitance. This nominal capacitance may vary in a given tolerance band and is usually temperature-dependent. The insulator (dielectric) between the two electrically conductive plate has finite breakdown voltage and resistance, hence the capacitor can tolerate only limited voltage and a charged capacitor itself discharges after a while if left alone. Dielectric properties can also be polarity-dependent. Due to physical construction parasitic inductances may also be present.

Properties of capacitors can be given by the following parameters:

- nominal capacitance,
- tolerance,
- maximum voltage (continuous),
- dissipation factor,
- insulator resistance.

Capacitance of a parallel-plate capacitor can be calculated using the following equation:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}, \quad (6-9)$$

where ε_0 is the dielectric constant, ε_r is the relative dielectric constant, A is the area of the plate and d denotes the distance of the plates (in the figure the gaps between dielectric and plates are only present to ensure visibility).

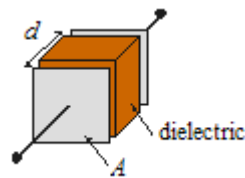


Figure 6–5 Parallel-plate capacitor

By analyzing the formula, some simple statements can be concluded. If we want to increase capacitance, dielectric with large dielectric constant, big plate surface area (ensuring place for more charges) or small plate distance is needed.

Capacitor model

The model of a real capacitor contains the inductance and the resistance of the terminals and the finite resistance of the dielectric (R_d).

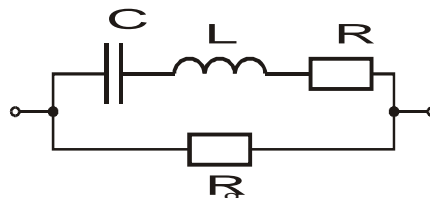


Figure 6–6 Model of the capacitor

Calculating the effective capacitance (considering only reactance) we can get the frequency-dependence of the capacitor's effective capacitance:

$$\frac{1}{j\omega C_{eff}} = \frac{1}{j\omega C} + j\omega L = \frac{1 - \omega^2 LC}{j\omega C}, \quad (6-10)$$

$$C_{eff} = \frac{C}{1 - \omega^2 LC}. \quad (6-11)$$

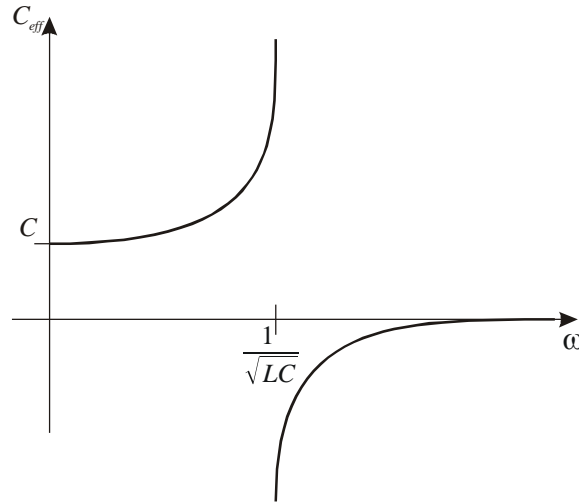


Figure 6–7 Frequency dependence of the effective capacitance

In ideal case the capacitance of the capacitor is frequency-independent, but as shown above, a capacitor behaves as a capacitance only in a limited frequency range, and the figure also shows that above the resonance frequency the capacitor can be considered as an inductive element.

The resistance of the dielectrics are high enough that in most practical cases it can be neglected. So the model is simplified to a series RLC circuit, which can be analyzed in a simpler way. It is practical to investigate the frequency dependence of some parameters – by this we can check the correctness of our model utilized during the measurement. The investigated parameters are the absolute value and the angle of the impedance and the variation of the effective capacitance and the dissipation factor in frequency domain. The impedance of the series RLC model is:

$$\bar{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j \frac{\omega^2 LC - 1}{\omega C}. \quad (6-12)$$

So the absolute value and the dissipation factor can be expressed as:

$$|\bar{Z}| = \sqrt{\left(R^2 + \left(\frac{\omega^2 LC - 1}{\omega C} \right)^2 \right)} = \frac{\sqrt{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}}{\omega C} \quad (6-13)$$

$$D = \frac{R}{\omega^2 LC - 1} = \frac{\omega RC}{\omega^2 LC - 1} \quad (6-14)$$

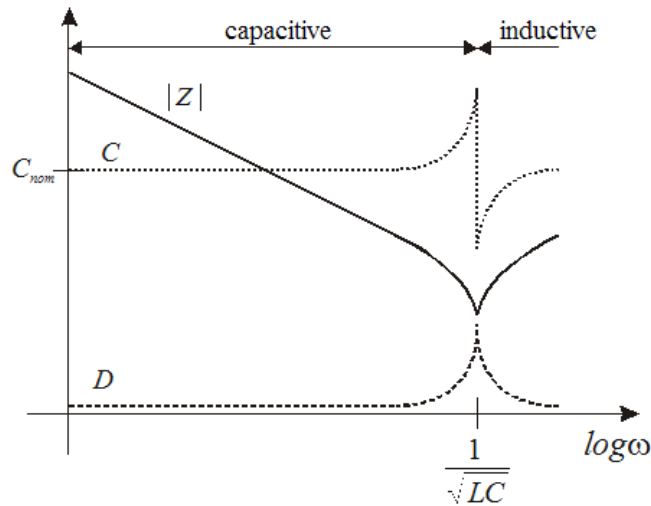


Figure 6–8 Theoretical frequency dependence of the capacitor model

Inductors

Inductors, also called coils, are components having certain inductance. Like capacitance, inductance may also vary in a given tolerance band.

Inductors can be characterized with the following parameters:

- nominal inductance,
- tolerance,
- maximum current (DC),
- quality factor,
- DC resistance.

Inductors are mostly solenoidal or toroidal (see Fig. 6–9). The self-inductance of a solenoid can easily be calculated using the following equation:

$$L = \frac{\mu_0 \mu_r n^2 A}{l}, \quad (6-15)$$

where μ_0 and μ_r are the vacuum and the relative permeability, respectively, n is the number of turns, A denotes the cross-sectional area and l is the length of the inductor. For toroidal inductors another simple (approximate) formula can be used:

$$L \approx \frac{\mu_0 \mu_r n^2 A}{2\pi a}, \quad (6-16)$$

where a is the inner radius of the toroid. It can be seen that by utilizing iron cores having high relative permeability the self-inductance can considerably be increased.

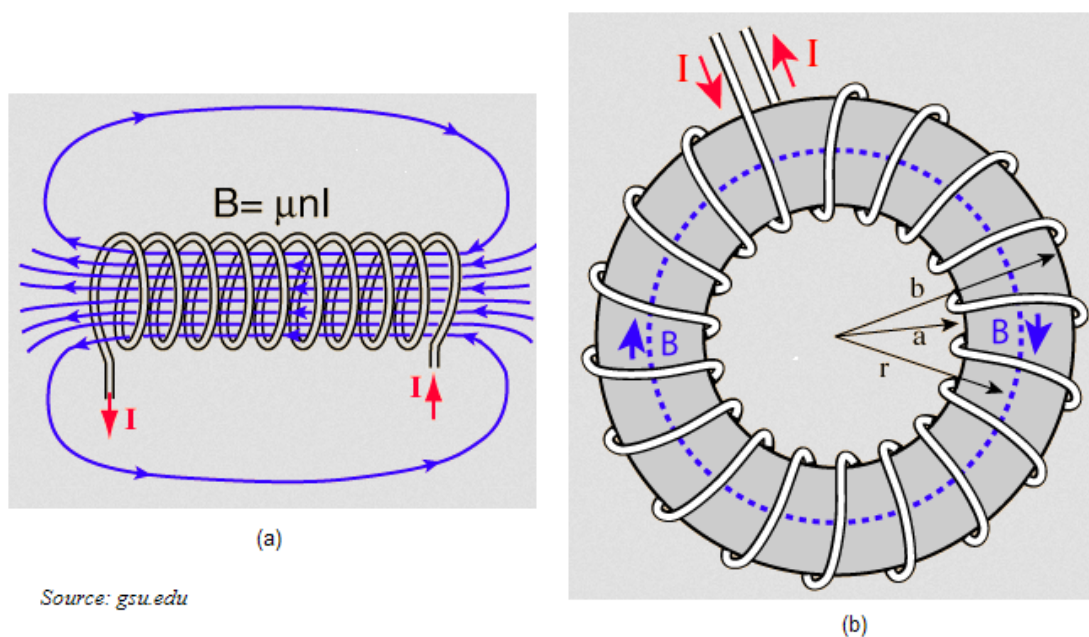


Figure 6-9 Solenoidal (a) and toroidal (b) inductors

Inductor model

The inductor model is quite similar to that of the capacitor's. In this model the parasitic elements are the resistance of the cable and the turn-to-turn capacitance (see Fig. 6-10). For more complex modeling iron loss should also be considered.

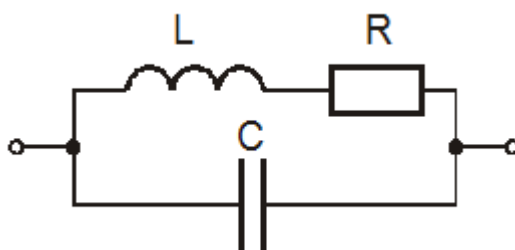


Figure 6-10 Model of the inductor

The frequency-dependence of inductors can be investigated similarly as shown in the previous section. It can be stated that inductors behave as inductance only in a limited frequency range. Using an iron core increases the inductance, but as a consequence, the resonance frequency decreases and the inductor can only be used (as an inductance) in a narrower frequency range.

Parallel LC

During this measurement exercise a parallel LC will also be measured. This circuit consists of an inductor and a capacitor. Due to non-idealities of the elements a resistance (R) should also be considered for more precise modeling, but for our further investigations this parallel resistance is neglected.

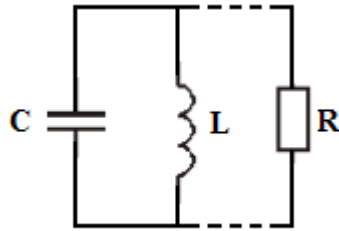


Figure 6–11 A parallel LC circuit

Let us determine the impedance of the LC part of the circuit:

$$Z = j\omega L \times \frac{1}{j\omega C} = \frac{L/C}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC} \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad (6-17)$$

$$\lim_{\omega \rightarrow 0} Z(\omega) = 0 \quad \lim_{\omega \rightarrow \infty} Z(\omega) = 0 \quad \lim_{\omega \rightarrow \omega_0} Z(\omega) = \infty. \quad (6-18)$$

At very low frequencies the inductor, while at very high frequencies the capacitor acts as a short-circuit in this compound two-terminal component. When measuring the resonance frequency of this simple system, a resistor should be connected in series with the LC circuit to protect the generator against short-circuits at these frequency ranges, and then a variable frequency voltage generator and an oscilloscope should be connected to the system. The generator frequency has to be adjusted so that the amplitude and the phase of the LC circuit approximately equals to that of the generator (at resonance frequency the circuit behaves more or less like an open circuit).

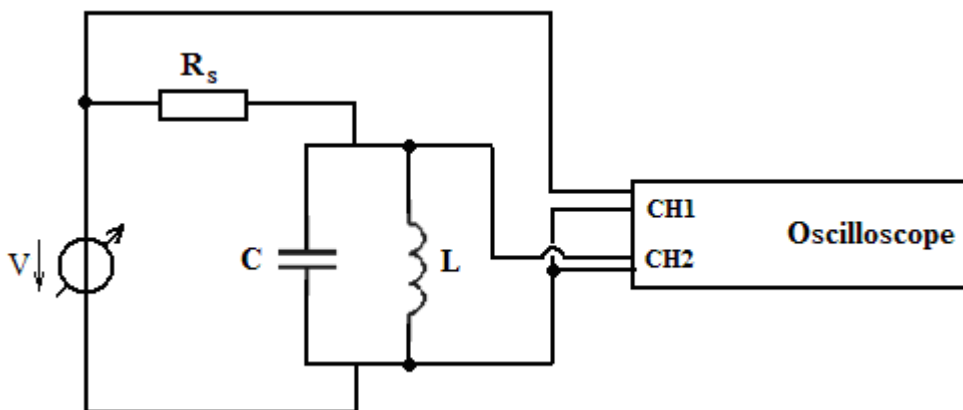


Figure 6–12 Resonance frequency measurement set-up

Preparation exercises

Before the measurement the following exercises are to be done independently.

1. Read the *Theoretical fundamentals of the measurement* carefully!
2. Calculate the impedance of a parallel LC circuit with loss!
3. Observe and consider the *Exercises*!
4. Answer the test questions at the end of this guide!

The preparation may be checked by oral or written questions, as well.

Measurement instruments

Impedance Analyzer

Wayne Kerr 6440 Component Analyzer

Test boards

The test boards contain the two-terminal components that will be used during the course. Moreover, components without mounting are available, too.

Exercises

1. Small resistance measurement

1.1. Measure the resistance of a 1-meter long cable with a high-precision multimeter using two- and four-wire methods as well, and then without using additional wires, i.e., connecting the cables into the multimeter directly. Repeat the measurement with the Wayne-Kerr impedance analyzer.

1.2. Make a coil from the cable being measured, first using standard winding, then bifilar winding. Plot the impedance characteristics for the standard and the bifilar wound coils, compare the results, and point out the differences.

2. Resistance as a function of power

Measure the non-ideal characteristics of a resistor, i.e., the dependence of its resistance on the temperature. Do not forget to check the voltage on the terminals. *Be careful: the resistor may become hot!*

3. Inductance measurements

Compare the impedance characteristics of an air core inductor with those of a similarly wound ferromagnetic core inductor. Explain the characteristic differences in details.

4. Resonance of a parallel LC circuit

4.1. Design a measurement setup to observe the resonance phenomenon and to measure the resonant frequency of a parallel LC circuit by *not* using the Wayne-Kerr impedance analyzer.

4.2. Measure the resonance parameters by using the Wayne-Kerr impedance analyzer. Plot the impedance characteristics.

5. Analyzing an unknown two-terminal circuit

Analyze the impedance of the two- or three-component circuit chosen by your instructor. Determine the components of this particular circuit as well as their actual parameters.

Test questions

1. What does it mean that a component (resistor, capacitor, inductor) is not an ideal one?
2. What is the quality factor and what is the damping ratio?
3. We get a negative capacitance value in the parallel RC model while trying to measure some impedance. Please explain this.
4. What measurement technique do impedance analyzers used in this laboratory utilize?
5. Explain why four-wire impedance measurement technique has advantages over the two-wire method in case of components with small impedance!
6. How can you calculate the resonant frequency of a parallel LC circuit knowing the values of the elements?
7. How can you calculate the quality factor of an LC circuit with loss?
8. How can you transform the parallel R-L model of an inductor into a series R-L model?
9. How can you transform the parallel R-C model of a capacitor into a series R-C model?
10. Assuming there is at least one decade between the cutoff frequencies, plot the Bode diagram of the components in Figure 6-13. Similarly, give the Bode-plots for the complex impedance of other compound two-terminals, built from two or three R/L/C tags.
11. Define the measurement setup to observe (and measure) the resonance phenomenon of a parallel LC-circuit with an oscilloscope!

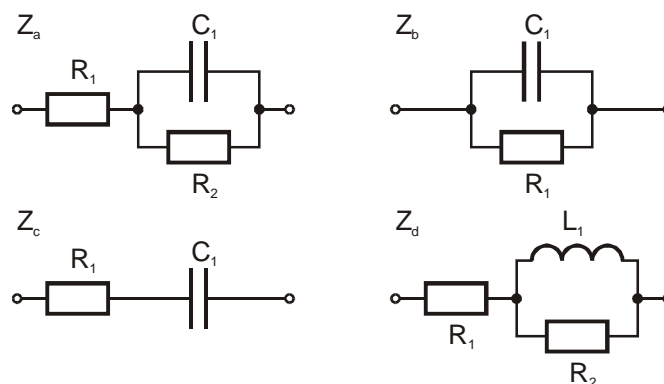


Figure 6-13 Models for unknown two-terminal circuits