

#### 4. N-DIM CASE

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$$\underline{y} = \underline{y}_n + \underline{e} \quad \underline{y}_n = \underline{\Phi} \underline{\theta}$$

$$\underline{e} \sim N(\underline{0}, \underline{\Sigma}) \quad \underline{y} \sim N(\underline{\Phi} \underline{\theta}, \underline{\Sigma})$$

$$\underline{y} \sim \underbrace{\frac{1}{(\sqrt{2\pi})^N |\underline{\Sigma}|} \exp \left[ -\frac{1}{2} (\underline{y} - \underline{\Phi} \underline{\theta})^T \underline{\Sigma}^{-1} (\underline{y} - \underline{\Phi} \underline{\theta}) \right]}_{L(\underline{\theta})}$$

$$\ell(\underline{\theta}) = -\frac{N}{2} \ln(2\pi) - \ln |\underline{\Sigma}| - \frac{1}{2} (\underline{y} - \underline{\Phi} \underline{\theta})^T \underline{\Sigma}^{-1} (\underline{y} - \underline{\Phi} \underline{\theta})$$

$$\frac{\partial \ell}{\partial \underline{\theta}} = \underline{\phi}$$

$$\frac{1}{2} \|\underline{\Sigma}^{-1/2} (\underline{y} - \underline{\Phi} \underline{\theta})\|_2^2$$

$$\hookrightarrow \left[ \hat{\underline{\theta}}_{ML} = \hat{\underline{\theta}}_{LS} = (\underline{\Phi}^T \underline{\Sigma}^{-1} \underline{\Phi})^{-1} \underline{\Phi}^T \underline{\Sigma}^{-1} \underline{y} \right]$$

EXAMPLE 2:  $\underline{\Sigma} = \sigma^2 \underline{I}$   $\underline{\Phi} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$  i.i.d.

$$\hat{\underline{\theta}}_{ML} = \underbrace{\left( [1 \dots 1] \frac{1}{\sigma^2} \underline{I} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \right)^{-1}}_{\sigma^2 \frac{1}{N}} \underbrace{[1 \dots 1] \frac{1}{\sigma^2} \underline{\Sigma}^{-1}}_{\frac{1}{\sigma^2} \sum y(n)} \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix} \rightarrow = \frac{1}{N} \sum_{n=1}^N y(n)$$

#### 5. OTHER PDF (EXAMPLE)

$$e(n) \sim \text{EXP}(0, \sigma^2)$$

$$y(n) = a + e(n)$$

$$y(n) \sim c e^{-|y(n) - a|}$$

$$L(a) \sim c^N e^{-\sum_{n=1}^N |y(n) - a|}$$

$$\frac{\partial \ell}{\partial a} = + \frac{\partial}{\partial a} \sum_{n=1}^N |y(n) - a| = \underline{\phi} \quad (\text{How?})$$

