

$$\left. \begin{aligned} \frac{\partial^2 \ell}{\partial a^2} &= -\frac{N}{\sigma^2} s^2 \\ \frac{\partial^2 \ell}{\partial a \partial b} &= \frac{\partial^2 \ell}{\partial b \partial a} = -\frac{N}{\sigma^2} \mu \\ \frac{\partial^2 \ell}{\partial b^2} &= -\frac{N}{\sigma^2} \end{aligned} \right\} \underline{\underline{M}} = \frac{N}{\sigma^2} \begin{bmatrix} s^2 & \mu \\ \mu & 1 \end{bmatrix}$$

$$\underline{\underline{CR(a,b)}} = \underline{\underline{M}}^{-1} = \frac{\sigma^2}{N} \frac{1}{s^2 - \mu^2} \begin{bmatrix} 1 & -\mu \\ -\mu & s^2 \end{bmatrix}$$

- IF  $\sigma^2 \uparrow$ , THEN  $CR \uparrow$
- IF  $N \uparrow$ , THEN  $CR \downarrow \sim \frac{1}{\sqrt{N}}$  ERROR DECAY
- CR DEPENDS ON  $\{t_n\} \rightarrow$  OPTIMAL EXPERIMENT DESIGN

$$- \sigma_a^2(a,b) = \frac{\sigma^2}{N(s^2 - \mu^2)} \geq \sigma_a^2(a) = \frac{\sigma^2}{N s^2}$$

MORE PARAMETERS  $\rightarrow$  LESS ACCURACY (HIGHER CR BOUND)

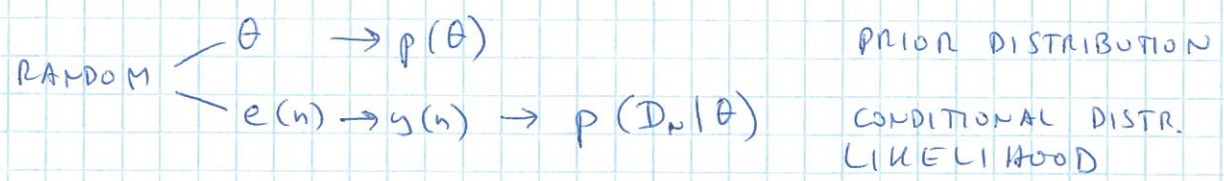
OVERFITTING

$\hookrightarrow$  REGULARIZED ML:

$$\hat{\theta}_{ML,R} = \arg \min_{\theta} [ \ell(\theta) + \lambda \|\theta\|^2 ]$$

$\hookrightarrow$  WHAT IS A GOOD VALUE?

7. BAYES APPROACH - MAXIMUM A POSTERIORI ESTIMATE



BAYES THEOREM:

DERIVATION

$$p(\theta | D_N) = \frac{p(D_N | \theta) p(\theta)}{p(D_N)} \sim p(D_N | \theta) p(\theta)$$

POSTERIOR DISTRIBUTION  $\sim$  LIKELIHOOD  $\times$  PRIOR